Dispersive and hyperbolic models of breaking waves and mixing on shallow water

Alexander Chesnokov

Novosibirk State University, Russia
email: chesnokov@hydro.nsc.ru

The wave breaking is a complex phenomenon accompanied by a strong vorticity generation and air entrainment. Breaking of water surface waves may occur anywhere that the amplitude is sufficient, however, it is particularly common on beaches because wave heights are amplified in the region of shallower water. If a wave approaching the coast is long and the variation of the coastal slope is gradual, spilling breakers usually appear. They are characterized by the presence of a finite turbulent zone spilling down the face of the wave (Longuet-Higgins & Turner 1974; Duncan 2001). At the toe of this turbulent zone, the wave slope changes sharply, resulting in flow separation and vorticity creation. The breaking waves entrain air into the water by forming 'whitewater' where an intensive dissipation occurs. Such a turbulent zone has a strong influence on the wave evolution.

The main goal of the talk is to present a method for constructing mathematical models of breaking waves. The proposed approach is based on a two-layer modelling, where the upper turbulent layer is considered within the framework of shear shallow water flows (Teshukov, 2007), while the lower layer is potential and can be described by the Green–Naghdi or Sen-Venan type models. The interaction between the layers is taken into account through a natural mixing process. Experimental data on the structure of the turbulent flow area under breaking waves show that the boundary between the turbulent and potential regions are clearly visible (Lin & Rockwell 1994; Misra et al. 2008).

The hydrostatic model of multi-layer flow proposed by Liapidevskii & Chesnokov (2014) correctly describes the vorticity generation and turbulent bores if the Froude number is sufficiently large. This model was generalized by Gavrilyuk, Liapidevskii & Chesnokov (2016) taking into account the non-hydrostatic (dispersive) effect in the lower layer. Both models neglect the air entrainment. In particular, the obtained dispersive model describes the transition from an undular bore to a breaking (monotone) bore when the Froud number is between 1.3 and 1.4. The shoaling and breaking of a solitary wave propagating in a long channel of mild slope are also well predicted by the model. It is in a good agreement with experimental data by Hsiao et al. Further (Gavrilyuk, Liapidevskii & Chesnokov, 2017) we extend our model taking into account the effect of air entrainment. As a result we derive a new two-layer model for the interaction between a bubbly shear layer and long internal waves over topography. We perform non-stationary calculations to describe the formation of bores, and both periodic and damped oscillations. A hydrostatic approximation of this model is compared to the full non-hydrostatic system. Even if a quantitative description by means of the both models is quite similar, the qualitative behaviour of the wave fronts is different. In particular, a non-monotonic behaviour of bores has been found by using the dispersive model, while it is monotonic in the hydrostatic model. The analogy between mathematical modelling of two-layer bubbly flows and internal waves is drawn. This allows us, although implicitly, to compare the theoretical and experimental results relating to the generation of internal waves of large amplitude (Nash & Moum, 2005; Harris & Decker, 2017).

This talk is based on the joint works with Sergey Gavrilyuk and Valery Liapidevskii.